THE ASSOCIATION OF TEACHERS OF MATHEMATICS IN THE MIDDLE STATES AND MARYLAND.

ALGEBRA SYLLABUS.

REPORT OF THE COMMITTEE.

ELEMENTARY AND INTERMEDIATE.

FOREWORD.

LIST OF TOPICS.

(NOTE.—In this list no suggestion of order of topics is intended.)

I. Extension of Arithmetic in Algebra. Positive and Negative Numbers. Definitions. Graphs.
II. Fundamental Operations.
III. Factoring.
IV. Highest Common Factor and Least Common Multiple by Factors.
V. Fractions; Reduction, Addition, Subtraction, Multiplication, and Division. Complex Fractions.
VI. Equations of the First Degree in One Unknown. Problems.
VII. Simultaneous Equations in Two and Three Unknowns. Graphs. Problems.
VIII. Involution and Evolution. Square Root of Polynomials and Arithmetical Numbers.
IX. Exponents and Radicals. Radical Equations.
X. Imaginaries.
XI. Quadratic Equations in One and Several Unknowns. Theory. Graphs. Problems.
XII. Binomial Theorem for Positive Integral Exponents.
XIII. Inequalities.
XIV. Ratio and Proportion.
XV. Progressions.

This syllabus is intended primarily for teachers. It specifies those topics which, in the opinion of the Association of Teachers of Mathematics in the Middle States and Maryland, should be included under the designation, *elementary and intermediate algebra.*

This association conceives one of the greatest purposes of the teaching of algebra in the schools to be the cultivation of the student's power in reasoning by helping him

1. To concentrate his mind, especially on a *system* of thought;
2. To generalize correctly; and,

3. To develop originality and insight by using skillfully a finely worked out language of symbols.

It is believed that the body of closely related truths in algebra are admirably adapted to the purpose just mentioned. This is not to say that examples and illustrations to provide interesting exercise in algebraic practice are not to be drawn from commerce, geometry, physics, mechanics; but such examples should be so simple as to require no extended explanation of their nature and whatever knowledge they imply should be regarded as incidental to the main purpose of cutivating the student's powers of reasoning. The first concern of the young student of algebra is the knowledge, logic, and operations contained within algebra itself, but it is perfectly possible to select attractive exercises which, while they do not carry the student far afield, will show him how algebra can be practically applied.

As the same idea occurs in different forms in various parts of the algebra, some repetition is unavoidable in the syllabus, but perhaps it is most desirable to recur several times to a method that does not change though it be applied to a form that is new.

Throughout the syllabus are notes bearing upon particular points, and at the close are appended a few general notes which are suggestive of the many observations the teacher has to make and of the caution his work constantly requires.

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I. Extension of Arithmetic in Algebra.

A. Literal numbers as the generalization of arithmetic numbers.

1. Indicated operations, $a+b$; $a-b$; $a \times b$; $a \div b$.
2. Powers and fractions resulting from indicated multiplications and divisions.
3. Negative numbers necessary for a more complete scale of numbers. In arithmetic $3-10$ is an impossible

operation. It becomes possible as soon as negative numbers are admitted. Introduce the scale . . . -3, -2, -1, 0, $+1$, $+2$, $+3$, . . . by addition and subtraction. Illustrate by divisions on a line, and by as many concrete examples as possible.

4. Simple problems involving the use of literal numbers, *e. g.*, John has a cents and received b cents, he spends x cents; how much has he? A letter stands for a number which may be integral, fractional, positive, or negative.

B. Simple equations.

1. Contrast 30% of cost = \$60 with $.3x = 60$.
   $5 + 7 = 12$, $19 - 3 = 22 - 6$; substitute a letter in these examples.
2. Solution of $3x - 4 = x + 8$ by the use of the equality axioms.
3. Discover law of transposition.
4. Literal equations.
   a. $x + a = b$
   b. $dx + b = c$
   c. More easy problems.
5. Some very simple problems resulting in numerical simultaneous equations.

Note 1. Extract definitions as they are needed.

Note 2. Graphs of simple forms such as

$$y = 2x + 1, \text{ and} \begin{cases} y = 2x + 1 \\ y = 3x + 2 \end{cases}$$

Use coördinate paper. Measure the x and y of intersection.

II. Fundamental Operations.

Note.—Introduce the laws of signs and of exponents and the laws of commutation, association, and distribution as applied to these operations. Some work in detached coefficients should be given.

A. Addition and Subtraction.

1. Algebraic addition involves arithmetic addition and subtraction.
2. Meaning of subtraction is to find a number which, added

to the subtrahend, gives the minuend. Check subtraction by addition.

3. Removal and introduction of signs of aggregation. Simple cases only.

B. Multiplication.

1. Monomials by monomials.
2. Polynomials by monomials.
3. Polynomials by polynomials.

Note 1.—Notion of *function* and *variable* may be suggested in this place by the evaluation of a polynomial for different values of the letter in it.

Note 2.—Laws of homogeneity may be pointed out.

4. Type products.

a. $(x \pm y)^2$, and the square of any polynomial.

b. $(x+y)(x-y)$, $(x+y+z)(x+y-z)$,
$(x^2+xy+y^2)(x^2-xy+y^2)$

c. $(cx+a)(cx+b)$

d. $(a \pm x)^3$

e. Of the form, $(a \pm b)(a^4 \mp a^3b + a^2b^2 \ldots)$.

Note.—It should be pointed out that the given expressions are the factors of the results.

C. Division.

1. Monomial divisors.
2. Polynomial divisors.
3. Special quotients of the type: $\frac{(x^n \pm y^n)}{x \pm y}$.

Note 1.—Binomial divisors and simple synthetic division are especially recommended at this point.

Note 2.—The Remainder Theorem may be introduced here by simple numerical examples. It may again be referred to in connection with factoring.

The simple proof by the identity $f(x) \equiv Q(x-a) + R$ may well be given here.

III. Factoring.

Note.—Refer to II. B, 4 and II. C, 3.

I. Common Factor.

  a. $ax + bx$.

  b. $a(x + y) + b(x + y)$

  c. Grouping, as $ax + ay + bx + by$.

2. Perfect square, $x^2 \pm 2ax + a^2$.
3. Difference of squares, $x^2 - a^2$, $x^2 \pm 2ax + a^2 - b^2$, $x^4 + x^2y^2 + y^4 = (x^4 + 2x^2y^2 + y^4) - x^2y^2$
4. Quadratic trinomial, $x^2 + ax + b$, $ax^2 + bx + c$.
5. Perfect cube, $x^3 \pm 3x^2y + 3xy^2 \pm y^3$.
6. Sum or difference of odd powers, $x^3 \pm y^3$.
7. Binomial factors by the remainder theorem or synthetic division. Simple cases only.

Note 1.—Above are the type forms to be led up to by many simple numerical examples.

Note 2.—Equations that can be solved by factoring, as $x^2 - 3x = 10$, $ax + bx + c = 0$, should be introduced here.

IV. Highest Common Factor and Lowest Common Multiple. [By Factoring.]

1. Monomials only:

   $60a^2x^2$, $45a^4x^3$, $20ax^4$.

2. Polynomials only:

   a. $x^3 - 8$, $x^2 - 4$, $x^2 + 5x - 14$.

   b. $x^2 - ax - bx + ab$, $x^2 - 2bx + b^2$, $x^2 - b^2$.

3. Products of monomial and polynomial.

   $10a^2x^2 - 90a^2$, $15a^3x^2 + 30a^2x - 225a^3$.

V. Fractions.

Note.—A whole number is a fraction with the denominator 1.

A. Reduction to lowest terms.

1. Monomial numerator and denominator:

$$\frac{49a^5x^3}{35a^7x^2} = \frac{7 \cdot 7 \cdot a^5 \cdot x^2 \cdot x}{7 \cdot 5 \cdot a^5 \cdot x^2 \cdot x^2} = \frac{7 \cdot a^5 \cdot a^2}{7 \cdot a^5 \cdot x^2} \cdot \frac{7x}{5a^2} = \frac{7x}{5a^2}.$$

Then show the cancellation method.

2. Polynomial numerator and denominator:

a. Type, $\frac{(x-7)(x+7)}{(x-7)(x-5)}$, $\frac{(x-a)(x+b)}{5(x-a)}$, etc.

b. Type, $\dfrac{(x-5)(x+3)}{a(5-x)}$, $\dfrac{(x-a)(x-b)}{(b-x)(b+x)}$

Note.—What is cancellation? Emphasize $a - a = 0$,

$$\frac{a}{a} = 1, \quad \frac{a-b}{a-b} = ? \quad \frac{a-b}{b-a} = ?$$

B. Reduction to mixed expressions and the reverse. Many examples, especially of the latter.

C. Addition and subtraction of fractions.

1. Reducing fractions to a common denominator.
    a. Monomial.
    b. Binomial and polynomial.
2. Addition and subtraction.
    a. Monomial denominators.
    b. Polynomial denominators.

(1) Type, $\dfrac{m+n}{m-n} - \dfrac{m-n}{m+n} - \dfrac{4m^2}{m^2-n^3}$

(2) Type, $\dfrac{b+c}{(a-b)(a-c)} + \dfrac{c+a}{(b-c)(b-a)} + \dfrac{a+b}{(c-a)(c-b)}$

Note.—Examples, in which the sum can be reduced to lower terms, are especially recommended.

D. Multiplication and division of fractions.

1. Monomial numerators and denominators.
2. Polynomial numerators and denominators.

Note 1.—Arithmetical processes with fractions such as multiplying and dividing both terms by the same number, reducing to the same common denominator, etc., should be recalled and compared with the corresponding operations in algebra. For each of these processes a sufficient reason should be given and appeal should be made to the student's intuition.

Note 2.—The first elementary notion of ratio may be introduced with fractions and the forms $\dfrac{a}{\infty}, \dfrac{0}{a}, \dfrac{a}{0}, \dfrac{0}{0}$, explained with such simple numerical examples as, $\dfrac{2}{\infty}, \dfrac{0}{2}, \dfrac{2}{0}, \dfrac{x^2-1}{x-1}$, when $x=1$.

E. Simplification of complex fractions.

1. The continued fraction (simple types only).

$$\cfrac{1}{1+\cfrac{1}{x+\cfrac{1}{x}}}$$

2. Complex fractions having a sum or difference in both numerator and denominator.

$$\frac{\dfrac{1}{a-b}-\dfrac{a}{a^2-b^2}}{\dfrac{a}{ab+b^2}-\dfrac{b}{a^2+ab}}$$

3. Mixed numbers in both numerator and denominator.

$$\frac{\dfrac{2mn}{m+n}-1}{1-\dfrac{n}{m+n}},\quad \frac{2x-1-\dfrac{10}{x-3}}{3x-5+\dfrac{12}{x-2}}$$

Note.—Multiplying both terms of the fraction by the same expression is often the simplest device. Examples should not be too complex and some should be workable in several ways.

VI. The Simple Equation.

A. Extended practice in the simple equation, resuming the consideration of transposition.

B. The fractional equation:

1. Monomial denominators:

$$\frac{x-1}{7}=7-\frac{4+x}{4}-\frac{23-x}{5}$$

2. Polynomial denominators:

a. $$\frac{2x+1}{3x-3}=\frac{7x+1}{6x-6}-\frac{2x^2-3x-45}{4x^2-4}$$

b. $$\frac{1}{x-1}-\frac{1}{x-2}=\frac{1}{x-3}-\frac{1}{x-4}$$

Note.—Simplify the members in *b* separately before clearing of fractions.

c. $$\frac{6x+1}{15}-\frac{2x-4}{7x-16}=\frac{2x-1}{5}$$

Note.—Care must be taken that extraneous roots are not introduced in clearing of fractions.

C. The literal equation:

1. Solution for one unknown.
2. "Formula work." Solving simple formulæ for each of the letters in terms of the others.

Note.—Only formulæ that can be ***briefly*** explained should be chosen, as:

a. $i = \frac{ptr}{100}$, (simple interest)

b. $d^2 = 2s^2$, (diagonal of a square)

c. $h = \frac{s}{2}\sqrt{3}$, (height of equilateral triangle)

d. $C = (F - 32)\frac{5}{9}$, (Centigrade scale)

e. $s = vt$

$c = 2\pi r$

$i = \frac{l}{d^2}$, (light), etc., giving numerical illustrations.

D. Problems leading to simple equations.

1. Oral work, as: the sum of a and $b = ?$ When a is divided by b, c is the quotient, and d the remainder; express algebraically. A train goes at the rate of m miles an hour for h hours, how far does it go? The interest on a dollars at b per cent. for c years is what?
2. Problems dealing with—
   a. Geometrical objects.
   b. Motion (trains, etc.)
   c. Number.
   d. Business.
3. Some simple abstract problems, illustrated by concrete examples.

VII. Equations in Two and Three Unknowns.

A. Two unknowns.

Elimination by, 1. Addition and subtraction.
2. Comparison.
3. Substitution.

Note.—Some equations should be solved by all three ways for the sake of comparing the methods.

B. Two unknowns, fractional form:

$$\frac{4}{5x}+\frac{5}{6y}=5\frac{11}{15}$$
$$\frac{5}{4x}-\frac{4}{5y}=\frac{11}{20}$$

C. Three unknowns.

1. Where all three equations contain all three unknowns; with small numerical coefficients only.
2. Where the three unknowns are not in each equation.

$$\frac{4}{x}-\frac{3}{y}=\frac{1}{20}$$
$$\frac{2}{z}-\frac{3}{x}=\frac{1}{15}$$
$$\frac{4}{z}-\frac{5}{y}=\frac{1}{12}$$

D. Literal equations.

$$(a-b)x-(a+b)y=-4ab$$
$$(a+b)x+(a-b)y=2a^2-2b^2$$

E. Problems leading to equations in two and three unknowns.

F. Plotting easy equations in two unknowns, such examples as give:

1. Lines intersecting at any angle.
2. At right angles.
3. Lines that are parallel, pointing out the relations between the coefficients.

VIII. Involution and Evolution.

A. Powers of a monomial. The general form $(a^m)^n$ carefully examined and illustrated.

B. Square, cube and fourth power of a binomial.

C. Definition of root. Principal root. Roots of a monomial.

D. Square root of:

1. Polynomial, presented through the trinomial.
2. Numbers.

Note.—Derive the general method by the type form $a^2+2ab+b^2$.

IX. Exponents and Radicals.

A. Exponents in fundamental operations. (Refer to II).

B. Theory of a positive integral exponent.

Theorems:

1. $x^a \cdot x^b = x^{a+b}$
2. $x^a \div x^b = x^{a-b}$ $\quad (a > b)$
3. $(x^a)^b = x^{ab}$
4. $(x^a)^b = (x^b)^a$
5. $(x^a y^b)^n = x^{an} y^{bn}$
6. $\left(\dfrac{x^a}{y^b}\right)^n = \dfrac{x^{an}}{y^{bn}}$
7. $(\sqrt[n]{x^a})^n = x^a$

Note.—In considering the above theorems it is sufficient to use numerical exponents, the exponents being the abbreviations of continued multiplication. Restrict a and b to positive integers, $a > b$.

C. Negative, zero, and fractional exponents:

Note.—Apply the principle of *no exception* to interpret these new exponential forms.

Theorems:

1. $\dfrac{a^m}{a^n} = \dfrac{a^{m-n}}{1} = \dfrac{1}{a^{n-m}} \quad \dfrac{1}{a^{-(m-n)}}$

The scale, . . . $a^{-4}, a^{-3}, a^{-2}, a^{-1}, a^0, a^1, a^2, a^3$ . . .

Work in fundamental operations with these exponents.

2. When $n = m$, $\dfrac{a^m}{a^n} = \dfrac{a^m}{a^m} = a^{m-m} = a^0 = 1$

3. a. $x^{\frac{a}{n}} = \sqrt[n]{x^a}$ *cf.* B. 7

b. $\sqrt[n]{x^a} \cdot \sqrt[n]{y^b} \cdot \sqrt[n]{x^a \cdot y^b}$

Note.—All these theorems should be first indicated by numerical examples.

D. *Note.*—Radicals should be closely associated with exponents wherever possible.

1. The fractional exponent and radical sign interchanged and compared in application.
2. The graphic representation of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$ by successive right-angled triangles.

E. Operations:

1. Removal of a perfect power: $\sqrt{18a^3b^5} = 3ab^2\sqrt{2ab}$
2. Reduction to lower degree: $\sqrt[6]{9a^2b^4} = \sqrt[3]{3ab^2}$, or vice versa.
3. Introduction of a factor under the radical sign: $2a\sqrt[3]{3b} = \sqrt[3]{24a^3b}$
4. The four fundamental operations and powers.

$$2\sqrt{a} + 3\sqrt{a} = 5\sqrt{a},\ \sqrt{a}\cdot\sqrt{b}\cdot\sqrt{c} = \sqrt{abc},\ \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}};$$

$(\sqrt{abc})^2$ etc. It is well to state some of these laws in words.

5. Rationalizing the denominator, limited to monomials and binomials of the second order, including such examples as:

$$\sqrt{\frac{x}{y}},\ \frac{\sqrt{3} - x}{\sqrt{2}},\ \frac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{\sqrt{1 + x^2} + \sqrt{1 - x^2}}$$

Find the approximate value of $\dfrac{\sqrt{5} - 1}{3 - 2\sqrt{5}}$ to three decimal places, *after* simplification.

6. Radical equations involving linear equations and quadratic equations having rational roots.

$$\sqrt{2x + 3} = 2x - 3$$
$$\sqrt{x - 1} + \sqrt{x + 4} = \sqrt{4x + 5}$$
$$a = \sqrt{\frac{b}{x}}$$
$$\frac{5x - 1}{\sqrt{5x} + 1} = 1 + \frac{\sqrt{5x} - 1}{1\frac{1}{3}}$$

Note.—All radical equations should be checked to insure that extraneous roots have not been introduced by squaring.

X. Imaginaries.

Note.—On entering upon the subject of imaginaries, it should be pointed out that negative numbers and fractions are not *real*

in the sense that numbers like 5 and $\frac{24}{6}$ are real, and hence in imaginaries the student is again extending his number system.

A. Definition equations.

1. $(\sqrt[n]{-a})^n = -a$

   Corollaries:

   a. $\sqrt{-a} = \sqrt{a}\sqrt{-1}$

   b. $\sqrt{-a}\sqrt{-b} = -\sqrt{ab}$

2. $\sqrt{-1} = i$

   1. Corollaries:

   a. $i^{4q+1} = i$

   b. $i^{4q+2} = -1$

   c. $i^{4q+3} = -i$

   d. $i^{4q+4} = +1$

Note.—Begin with $q = 0, 1, 2,$ etc. The constant use of the symbol i is strongly recommended.

B. Complex numbers.

1. Definitions.

2. Theorems.

   a. The sum and product of two conjugate complex numbers are both real.

   b. If $X + iY = 0$, then $X = 0$ and $Y = 0$

   Corollary:

   If $X + iY = A + iB$, then $X = A$ and $Y = B$.

3. The fundamental operations.

   a. Addition and subtraction.

b. Multiplication and division.

   Rationalization of denominators.

   Conjugates.

C. Graphs of complex numbers; their sums and differences.

XI. Quadratic Equations in One and Several Unknowns.

A. Equations in one variable.

1. Solution.

   a. Incomplete.

   (1) Pure, reducible to the form $x^2 = a^2$, $x = \pm a$; or $x^2 - a = 0$, $(x + \sqrt{a})(x - \sqrt{a}) = 0$, etc.

(2) Lacking the independent term, of form $ax^2 + bx = 0$. Solved by factoring.

b. Complete, reducible to the form $ax^2 + bx + c = 0$

Solved by 1. Factoring.

2. Completing the square.
3. Quadratic formula.

Note.—Fractional and radical equations which reduce to these type forms are included. Care should be taken that clearing of fractions or clearing of radicals does not introduce a false root.

c. Equations that are quadratic in a power of the variable or any function of it.

(1) In a power, as $ax^{2n} + bx^n + c = 0$

or $ax^{\frac{m}{n}} + bx^{\frac{m}{2n}} + c = 0$

(2) In any function, as

$$(x^2 + 2x)^2 - 2(x^2 + 2x) - 3 = 0,$$
$$2x^2 - 6x + \sqrt{x^2 - 3x + 6} - 9 = 0,$$
$$\frac{x^2}{x+1} + \frac{x+1}{x^2} = \frac{5}{2}.$$

Solved by factoring, or by substituting a new variable for the function involved.

d. Equations of higher degree that factor by known methods, especially by synthetic division.

2. Theory.

a. Proof of the quadratic formula.

b. Symmetric functions of the roots:

(1) Sum and product of the roots; application to testing results and to forming equations from given roots. Simple symmetric functions, as $\frac{1}{r} + \frac{1}{s}$, where r and s are the roots.

(2) Use of the product to show the roots are reciprocal, if the coefficient of the second degree term equals the independent term.

c. Discriminant test for the roots:

(1) Real or imaginary.
(2) Rational or irrational.
(3) Equal or unequal.

Note.—Applications such as the following are recommended: For what values of a will the roots of $2x^2 + (1 + a)x + 2 = 0$ be equal? Real?

d. Graphs. Type form, $ax^2 + bx + c$.
(1) Let $ax^2 + bx + c = y$.
(2) For approximation of the roots, change to form $x^2 + px + q$, let $y = x^2$ and $y + px + q = 0$. Plot the intersections of the curve and the straight line. The curve $y = x^2$ is convenient for all numerical quadratic equations.

e. Problems including numerical, geometrical, physical and commercial relations, provided their subject matter does not require extended explanation.

B. Equations in two variables that can be solved by quadratic methods.

1. Solution.

a. When one is of the first degree, the other of second degree or factorable by the simple equation so as to reduce to second degree.
Solved by substitution, as
$ax + by = c$
$dx^2 + exy + fx = k$, etc., etc.

b. Homogeneous second degree equations.
(1) One entirely homogeneous, the other not, as
$ax^2 + bxy + cy^2 = 0$
$lx^2 + y + my^2 = k$
Solved by factoring, then substituting.
(2) Both homogeneous except for the independent term, as
$ax^2 + bxy + cy^2 = d$
$ex^2 + fxy + gy^2 = k$
Solved by eliminating the independent term, then treating like case (1).

c. Symmetric forms (or symmetric but for sign)
Solved by obtaining $x + y$ and $x - y$

(1) Of types $x+y$, $x-y$, xy, x^2+y^2,

$x^2 \pm xy + y^2$, $x^4+x^2y^2+y^4$, $\frac{x}{y}+\frac{y}{x}$

(2) Of type $(ax)^n+(by)^n=k$

$ax+by=1$

Solved by elimination of the powers. Divide if possible.

d. Other methods.

(1) Divide one equation by the other.

(*a*) When it divides evenly, as

$$x^3+y^3=28$$
$$x+y=4$$

(*b*) When a like factor appears in a member of each equation, as

$$a(x^2-y^2)=b$$
$$c(x-y)=d$$

(2) Add or subtract:

(*a*) When one unknown is thus eliminated, as

$$x^2+y=7$$
$$x^2-x-y=1$$

(*b*) When the result is a factorable form, as

$$x^2+xy+y^2+x+y=10$$
$$xy+2x+2y=8$$

(3) Factor when possible, as

$$x^2y^2 \pm 5xy+4=0$$
$$x^2+y^2-xy=13$$

e. Equations of the foregoing types which are in terms of functions of the variables, as

$$(x+y)^2+x^2y^2=13$$
$$x+y-xy=1$$

Solve for the functions (here, for $x+y$ and xy).

Such forms may be simplified by the substitution of a new variable.

2. Graphs.

Of any equation in two variables.

Approximation of results for cases that do not solve by quadratic methods. Simple cases only.

3. Problems.

Resulting in equations of the foregoing types.

C. Equations in three or more variables.
Simple types only.

XII. BINOMIAL THEOREM FOR POSITIVE INTEGRAL EXPONENTS.

A. Theory.
1. The derivation of the expansion of $(a+b)^n$ by simple inductive reasoning.
2. Discussion of the formula.
a. As a whole.
(1) Number of terms.
(2) Equality of coefficients of terms equally distant from the extremes.
b. The general term.
(1) Form of the coefficient in the fractional factored form.
(2) Sum of exponents in any term.

B. Applications.
1. To expansion of any binomial.
2. To finding any term, by formula or by analogy, as with the 3d term.
3. Such problems as, *a.* Find coefficient of x^{12} in $(x^2+2x)^{10}$
b. Find term independent of x in $\left(x^2-\frac{1}{x}\right)^{12}$
c. Find middle term of $\left(2-\frac{x}{3}\right)^8$
4. Powers of numbers, as $(1.1)^{12}=(1+.1)^{12}$ correct to 3 decimal places.

XIII. INEQUALITIES.

A. Recognition of the following principles:
1. Unequals combined with equals.
a. (1) Equals added to unequals give unequals in the same sense.
(2) Equals taken from unequals give unequals in the same sense.
(3) A term can be changed from one member of an inequality to the other if the sign of the term

be changed; application to annulment of like terms in both members.

(4) The signs of all terms of an inequality can be changed if the inequality sign is reversed.

b. (1) Positive equals multiplied into unequals give unequals in the same sense.

(2) Positive equals divided into unequals give unequals in the same sense.

c. (1) Unequals taken from equals give results unequal in the opposite sense.

(2) Positive unequals divided into positive equals give results unequal in the opposite sense.

2. Unequals combined with unequals.

a. Unequals added to unequals in the same sense give results unequal in that sense.

b. (1) Positive unequals multiplied by positive unequals in the same sense give results unequal in that sense.

(2) Positive unequals raised to a positive integral power give results unequal in the same sense.

(3) Positive unequals raised to a positive fractional power give results unequal in the same sense.

(4) Positive unequals raised to a negative power give results unequal in the opposite sense: reciprocals.

Note.—Unequals should not be taken from or divided by unequals, for the results can not in general be determined.

B. Solutions.

One unknown.

a. Of type $ax^2 + bx + c \gtreqless 0$

b. Of type $\dfrac{(x-a)(x-b)}{(x-c)(x-d)} \gtreqless 0$

Note.—Graphs can be used to show the values of x that make the function greater than, equal to, or less than, zero.

C. Use of the theorems, with the fact that a perfect square cannot be negative, to prove, (a and b being positive unequal numbers), $a^2 + b^2 > 2ab$, $a^3 + b^3 > a^2b + ab^2$, etc.

XIV. Ratio and Proportion.

A. Proofs of the following theorems:

1. *a.* The product of the means equals the product of the extremes.

   b. In a mean proportion, the mean equals the square root of the product of the extremes.

   c. Either mean equals the product of the extremes divided by the other mean, and either extreme equals the product of the means divided by the other extreme.

2. *a.* If the product of two quantities equals the product of two other quantities, either pair may be made the means, and the other pair the extremes, of a proportion.

   b. If four quantities are in proportion, any proportion of these quantities is true if the means of the original proportion are either both means or both extremes, in the new proportion.

3. *a.* If four quantities are in proportion, they are in proportion by composition.

   b. If four quantities are in proportion, they are in proportion by division.

   c. If four quantities are in proportion, they are in proportion by composition and division.

4. *a.* If four quantities are in proportion, equi-multiples (integral or fractional) of the antecedents are in proportion to equi-multiples of the consequents.

   b. If four quantities are in proportion, like powers of those quantities are in proportion.

5. If a series of ratios are equal, the ratio of the sum of their antecedents to the sum of their consequents equals any one of those ratios.

B. Applications.

1. Deduction of any proportion from a related proportion.
2. Finding one term if the others are known.
3. Solving equations (usually by composition and division).

XV. Progressions.

A. Arithmetic.

1. Deduction of formulas for

a. General or last term.
b. Sum of any number of terms.
c. Mean between two quantities.

2. Applications.
    a. Given any three of the five numbers, first term, difference, number of terms, last term, sum, to find the other two.
    b. Insertion of any number of means between two quantities.
    c. Problems of other types that can be solved by use of the formulas.

B. Geometric.

1. Deduction of formulas for
    a. General or last term.
    b. (*a*) Sum of any number of terms.
        (*b*) Limit approached by the sum of an infinite decreasing series.
    c. Mean between two quantities.
2. Applications.
    a. Given any three of the five numbers, first term, ratio, number of terms, last term, sum, to find the other two, unless the equations cannot be solved by known methods.
    b. Insertion of any number of means between two quantities.
    c. Evaluation of recurring decimals.
    d. Problems of other types that can be solved by use of the formulas, some combining the two progressions, and some reviewing important types of quadratics.

General Notes.

1. All the topics in this syllabus may at first be studied in outline with simple examples and then reviewed in detail with harder examples.

2. There should be much practice in mental algebra. Students should learn to give the elementary, simple forms accurately and quickly.

3. Students should state principles in general language with-

out symbols and without writing; at other times the general principle should be given them and they should be required to translate it promptly into symbolic language.

4. The definition should be given at the moment that it is required. It should be more often evoked than imposed. The student may be allowed to make and to correct his own statement. A temporary definition may be allowed until such time as correction and enlargement are necessary.

5. Home examples should relate generally to what has already been studied in class. The student should be encouraged to select for himself examples belonging to any advance work he is preparing. Sometimes a special topic in which a student has made commendable progress may be assigned to that student for further work which he may explain to the class.

6. Clear, orderly expression, writing and arrangement should be exacted at all times. The student should be made to feel the disadvantage of disorderly arrangement in expression.

7. Guessing in answering should be discouraged. A student should only say what at least he believes to be correct, and he should not lay too much emphasis on answers merely as answers. He should distinguish between answers and roots in certain problems.

8. Great care should be taken that algebraic work does not become mechanical. It may be necessary to repeat many times the *meaning* of the various algebraic forms.

9. The equation is the chief concern of algebra. In performing the various operations on equations great care should be taken with signs, with interchange of members, with simplifications sometimes before and sometimes after clearing of fractions, etc. Care should be taken to distinguish between unknown and variable. In the simple graphs of equations with one unknown, and of simultaneous equations with two unknowns, very clear distinction of the lines representing the values of the unknown or unknowns should be made.

10. In literal equations the meanings of the several kinds of letters used should be emphasized and generously illustrated by numerical values.

11. Operations should be frequently checked, preferably by numerical substitution wherever possible.